Applied Soft Computing Journal 88 (2020) 106061

Contents lists available at ScienceDirect

Applied Soft Computing Journal

journal homepage: www.elsevier.com/locate/asoc

Application of Fuzzy Reference Ideal Method (FRIM) to the military advanced training aircraft selection

J.M. Sánchez-Lozano *, O. Naranjo Rodríguez

University Centre of Defence at the Spanish Air Force Academy, MDE-UPCT, San Javier, Spain

ARTICLE INFO

Article history:
Received 9 July 2019
Received in revised form 24 November 2019
Accepted 28 December 2019
Available online 3 January 2020

Keywords:
Advanced training aircraft
Multi-Criteria Decision Making (MCDM)
Fuzzy logic
Analytic Hierarchy Process (AHP)
Reference Ideal Method (RIM)

ABSTRACT

When a nation needs to acquire a new military training aircraft for its Air Force, many factors must be taken into account. This requires a good command of conflicting factors, which can benefit from the domain of Multi-Criteria Decision Making (MCDM). However, some criteria involved in the assessment process are often imprecise or vague and the use of linguistic terms characterized by fuzzy numbers could be advisable. The aim of this research is thus to extract the best of a combination of Fuzzy MCDM approaches with the aim of solving a real decision problem of interest for the Spanish Air Force, specifically, the selection of the best military advanced training aircraft, based on a set of criteria of differing natures. This decision problem involves, on the one hand, quantitative or technical criteria (combat ceiling, operational speed, take-off race, etc.) and, on the other hand, qualitative criteria (maneuverability, ergonomics, etc.) based on the experience of a set of flight instructors of the 23rd Fighter and Attack Training Wing, collected via questionnaires. The Analytic Hierarchy Process (AHP) is applied to obtain the weights of the criteria, whereas the Reference Ideal Method (RIM) and its Fuzzy version (FRIM) are used to evaluate the alternatives based on a reference ideal alternative defined by the flight instructors mentioned above. As a result, the Italian Alenia Aermacchi M-346 Master aircraft was selected as the best option.

1. Introduction

The acquisition of a military aircraft is a complex task in which multiple factors must be taken into consideration [1]. From the point of view of analyzing the best candidate, an aircraft which has good qualities is not always the best option. In these cases, factors such as business strategies across nations, compatibility with other platforms, maintenance costs, etc. are also relevant [2]. Nowadays, most military aircraft fulfill the main requirements and technical standards and thus it is unlikely that any particular aircraft would stand out from the others. In fact, technical features have become secondary in detriment of qualitative and subjective criteria, which are becoming the key factors. Therefore, the coexistence of factors of different natures (quantitative and qualitative criteria) enables to take into account not only the main technical features provided by the manufacturers, but also subjective criteria such as compatibility, maneuverability, ergonomics, etc. which can be analyzed through the judgment of an experts system.

Due to that fact, the combination of Multi-Criteria Decision Making (MCDM) methodologies with techniques such as fuzzy logic provides a very useful way of dealing with this type of decision problems. The literature provides many examples of MCDM applications in the military field [3–9] and, their combination with fuzzy logic is also being analyzed in recent decades. Table 1 presents some military studies which combine MCDM methodologies and fuzzy logic. With reference to that table, it should be noted that the use of the Analytic Hierarchy Process (AHP) [10] in its fuzzy version (FAHP) [11,12] is very widespread. From the perspective of evaluating aircraft, this MCDM approach is combined with the fuzzy version of the TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) method [13,14].

The combination of both methods with fuzzy logic (Fuzzy AHP or Fuzzy TOPSIS) has been extensively used in a huge number of MCDM applications [15]. The reasons for this include their ability to integrate the analysis of quantitative and qualitative variables with the aim of providing an appropriate language to handle imprecise criteria [16]. Furthermore, due to the difficulty of the AHP methodology to consider a high number of criteria and alternatives, the number of pairwise comparisons carried out by the decision maker should remain below a reasonable threshold [17]. By combining it with the TOPSIS method enables us to apply the AHP methodology only to obtain the weights of the criteria, without the need to perform a large number of pairwise comparisons. However, the mathematical foundations of TOPSIS mean that it is not particularly suitable for solving

* Corresponding author.
E-mail address: juanmi.sanchez@cud.upct.es (J.M. Sánchez-Lozano).

https://doi.org/10.1016/j.asoc.2020.106061
1568-4946/

List of abbreviations and acronyms

MCDM	Multi-Criteria Decision Making
AHP	Analytic Hierarchy Process
FAHP	Fuzzy Analytic Hierarchy Process
RIM	Reference Ideal Method
FRIM	Fuzzy Reference Ideal Method
TOPSIS	Technique for Order Preference by Similarity to Ideal Solution
VIKOR	VIseKriterijumska Optimizacija I Kompromisno Resenje
CI	Consistency Index
CR	Consistency Ratio
RI	Random Index
MALOG	Logistics Support Command of the Spanish Air Force
EMA	Air Staff of the Spanish Air Force
KAI	Korean Aerospace Industries
FBW	Fly-by-wire
AoA	Angle-of-attack
ROKAF	Republic of Korea Air Force
MFD	Multifunction display
HUD	Head-Up Display
FOD	Foreign Object Damage
NATO	North Atlantic Treaty Organization
WSM	Weighted Sum Model

decision-making problems in which the ideal alternative does not require maximum or minimum criteria. Clear proof of that is in the service ceiling and cruising speed criteria for the case of the military training aircraft assessment [18,19]. These two criteria should be above a certain value, without necessarily reaching the maximum value.

Therefore, although the TOPSIS method has rational and understandable logic and its computation processes are straightforward [18], it is based on the concept of positive ideal solution and negative ideal solution, where the criteria which have an influence on the decision problem are criteria to maximize or minimize. On certain occasions, as mentioned above, one or several criteria may not need to have the maximum or minimum value, but that said criteria should fall within a range of values. In such cases, the Reference Ideal Method (RIM) [20] is an ideal application. As with the TOPSIS method, the RIM approach is based on the concept of ideal alternative; however, unlike other MCDM methodologies such as the TOPSIS and VIKOR (VIseKriterijumska Optimizacija I Kompromisno Resenje) methods [21], the RIM approach enables to evaluate alternatives without the need for the ideal values of the criteria to be maximums or minimums, said values can even belong to an interval. Furthermore, its fuzzy version (Fuzzy RIM or FRIM) developed recently [22], is particularly useful when qualitative and quantitative criteria coexist, i.e. to solve problems in which there is vagueness or imprecision in the data and which therefore must be expressed as fuzzy numbers. The assessment of military training aircraft involves criteria with different natures. Criteria such as endurance, cruising speed or landing distance are quantitative in nature, while tactical capability, ergonomics or maneuverability have a qualitative nature.

For all these reasons, we have chosen the RIM approach to solve the selection problem of military advanced training aircraft for the Spanish Air Force. Furthermore, in the proposed decision problem, the AHP methodology will also be used in order to

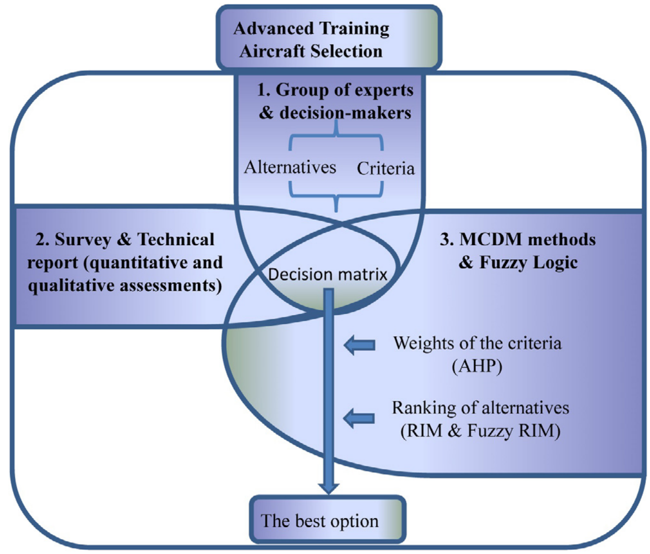

Fig. 1. Process schema. First, a group of experts and decision-makers from the Spanish Air Force will establish the alternatives and criteria. Secondly, quantitative and qualitative assessments of the alternatives and criteria (decision matrix) will provide through survey and technical report. Subsequently, the AHP methodology will allow to obtain the weightings of the criteria that influence the decision. Finally, the RIM and FRIM methodologies will enable the evaluation of the alternatives on the basis of quantitative and qualitative criteria.

determine the weights or coefficients of importance of the criteria that influence the valuation of each of the alternatives.

Therefore, from the literature, the combination of the RIM approach (and its Fuzzy version, FRIM) with AHP has never been carried out to date, but also that such a combination extracts the best of both MCDM methodologies, taking advantage of the main qualities of each technique. Therein lies the novelty regarding the present article. In fact, the present study allows to demonstrate how a combination of approaches of different multi-criteria methods (AHP, RIM and FRIM, in this case) leads to solving a current decision problem for the Spanish Air Force, that of military advanced training aircraft selection (Fig. 1).

This paper is divided into five parts: the second part describes the methodology used for the decision problem considered, i.e., the AHP methodology and the RIM approach (and its Fuzzy version FRIM); these MCDM methods are applied to solve the proposed problem; in the third part, the proposed decision problem (via the description of its alternatives and criteria) is presented, analyzed and discussed; in the fourth part, a sensitivity analysis of the results obtained is performed, and finally, the fifth part contains the main conclusions of this study.

2. Methodology

2.1. Analytic Hierarchy Process (AHP)

The AHP constitutes a robust and flexible MCDM approach to tackle complex decision-making problems [10]. The main goal of AHP is to allow the decision maker to determine the influence of each variable in a hierarchy process.

The three main objectives of AHP are as follows:

i. To structure complex decisions as a hierarchy of goals, criteria, and alternatives.
ii. To conduct a pairwise comparison of all the elements in each level of the hierarchy with respect to each criterion in the previous level of the hierarchy.

Table 1
Summary of military studies that combine fuzzy logic and MCDM methodologies.

Military application	MCDM method	Reference
Evaluating weapon system (artillery models)	AHP	[23,24]
Evaluating naval tactical missile systems	AHP	[25,26]
Evaluating guided missile destroyer	Catastrophe series	[27]
Evaluating weapon system (three missile systems)	Linguistic AHP	[28]
Evaluating attack helicopters	AHP	[29]
Evaluating initial training aircraft	TOPSIS	[18]
Assessment of military aircraft	AHP and TOPSIS	[30]
Selecting weapon system (five infantry rifles)	AHP and TOPSIS	[17]
Assignment of personnel	Ideal/Anti-Ideal concepts algorithm	[31]
Selecting Unmanned Aerial Vehicle (UAV)	AHP	[32]
Evaluating Offset Transaction Policy in Taiwan	AHP	[33]
Evaluating military training aircrafts	AHP and TOPSIS	[19]
Selecting transport mode (Taiwan off-shore islands)	AHP	[34]

iii. To vertically synthesize judgments on different levels of the hierarchy.

Note that AHP was applied in this study to calculate the weights w_i of the criteria based on information provided by a group of experts surveyed by the authors. Herein, the basics of the AHP methodology are outlined. The quantified judgments provided by the decision maker for a pair of criteria (C_i, C_j) are arranged into an n-order matrix, C. For instance, the matrix entry c_{12} refers to the relative significance of criterion C_1 with respect to C_2, namely, $c_{12} \approx (w_1/w2)$. Hence, the following rules govern how the AHP methodology works:

- $c_{ij} \approx (w_i/wj)$ for all $i, j = 1, 2, \ldots, n$.
- $c_{ii} = 1$ for all $i = 1, 2, \ldots, n$.
- If $c_{ij} = \alpha \neq 0$, then $c_{ji} = 1/\alpha$ for all $i = 1, 2, \ldots, n$.
- If criterion C_i is more important than C_j, then $c_{ij} \cong (w_i/wj) > 1$.

Note that the above conditions lead to C being a positive and symmetric matrix with 1s on the main diagonal. Thus, the judgments the decision maker provided fill in the upper triangle of matrix C. The values assigned to each entry of C usually lie on the interval [1,9] or their reciprocals, according to Saaty's scale [35]. The decision maker's linguistic preferences for carrying out a pairwise comparison are as follows. First, a value equal to 1 on Saaty's scale—labeled EI—denotes that both criteria C_i and C_j are *equally important*. A value on Saaty's scale equal to 3 (resp., 1/3) means that criterion C_i is *weakly more important* than C_j and has been labeled as WMI. SMI denotes that criterion C_i is *strongly more important* than C_j and corresponds to a value on Saaty's scale equal to 5 (resp., 1/5). The label VSMI means that C_i is *very strongly more important* than C_j and corresponds to a value on Saaty's scale equal to 7 (resp., 1/7). Finally, a value of 9 on that scale (resp., 1/9) denotes that criterion C_i is *absolutely more important* than C_j and has been assigned the label AMI.

If n is the order of matrix C, then the number of judgments in the corresponding upper triangular matrix is given by $L=n(n-1)/2$.

The eigenvector related to the maximum eigenvalue, λ_{max}, of C gives the list of weights. The standard eigenvector method to estimate the criteria weights measures the consistency and robustness of the referee's preferences, arranged in a comparison matrix via the Consistency Index (CI), calculated as follows: $CI = (\lambda_{max} - n)/(n-1)$ [36,37].

Thus, if the expert shows a minor inconsistency, it holds that $\lambda_{max} > n$. Saaty's scale provides the following metric of the so-called Consistency Ratio: $CR = CI/RI$, where RI is the Random Index, which can be calculated as the average value of CI for random matrices [36]. Accordingly, CI allows for the quantification of the probability that the matrix containing the judgments provided by the experts has been created randomly. Thus, C is said to be consistent (according to Saaty's scale) provided that $CR < 0.1$.

In this work, the AHP methodology enables the weights or coefficients of importance of the criteria which have influence in the phase of alternatives assessment to be obtained.

2.2. The Reference Ideal Method (RIM)

The RIM approach [20] is a novel MCDM methodology allowing information aggregation for a reference ideal. Such a procedure presents a compensatory nature in the following sense: it takes all the criteria into account simultaneously and allows a criterion value close to the corresponding ideal value to be weighted to the detriment of the value of another criterion distant from its ideal value. Interestingly, this allows for an expanded range of TOPSIS applications for decision-making problems with ideal solutions lying between the minimum and maximum values. The steps of the RIM approach are outlined below.

Step 1. Define the context of the work. In this stage, the conditions of the context of the work are established, and the following specifics for each criterion are identified:

- The range, t_j, being an interval, a set of labels, or a set of values that belong to a certain domain D.
- The reference ideal, s_j. This is an interval, a set of labels, or a set of values that represents the maximum importance in a given range. The reference ideal can be a set between the minimum and maximum values or a point.
- The weight of each criterion, w_j.

Step 2. Calculate the $m \times n-$ valuation matrix $X = (x_{ij})_{i,j}$ for $i = 1, \ldots, m$ and $j = 1, \ldots, n$, according to the set of criteria in the MCDM problem.

Step 3. Normalize the valuation matrix X by the reference ideal. Thus, for $i = 1, \ldots, m$ and $j = 1, \ldots, n$, the $m \times n-$ matrix $Y = (f(x_{ij}, t_j, s_j))_{i,j}$ must be calculated, where the piecewise function f: $x \oplus [A,B] \oplus [C,D] \rightarrow [0,1]$ is defined as follows:

$$f(x, [A, B], [C, D]) = \begin{cases} 1 & if \quad x \in [C, D] \\ 1 - \dfrac{d(x, [C, D])}{|A - C|} & if \ x \in [A, C] \ with \ A \neq C \\ 1 - \dfrac{d(x, [C, D])}{|D - B|} & if \ x \in [D, B] \ with \ D \neq B \end{cases} \quad (1)$$

It should be mentioned here that $[A, B]$ is a range that belongs to the universe of discourse $[C, D]$, and represents the reference ideal, $x \in [A, B]$ and $[C, D] \subset [A, B]$. In addition, the distance to the reference ideal $[C, D]$ is given by

$$d(x, [C, D]) = min\{|x - C|, |x - D|\} \quad (2)$$

where x is the value of a certain approach. Note that f always gives values lying within the range [0, 1]. Thus, the behavior of function f can be understood as follows. If the values in the preimage of f are equal to 1, then they match the reference ideal. However, the more distant they are from 1, the more distant they are from the reference ideal, independently of the evaluated variables. Thus, f is a normalization function described in terms of both the range and the reference ideal.

Step 4. Calculate the weighted normalized matrix $Y' = Y \otimes W = (y_{ij} \cdot wj)_{ij}$ for $i = 1,\ldots,m$ and $j = 1,\ldots,n$.

Step 5. Determine the variation of each alternative with respect to the normalized reference ideal. To do so, let $I_i^+ = \left\{\sum_{j=1}^{n}\left(y'_{ij} - w_j\right)^2\right\}^{\frac{1}{2}}$ and $I_i^- = \left\{\sum_{j=1}^{n}\left(y'_{ij}\right)^2\right\}^{\frac{1}{2}}$ for $i = 1,\ldots,m$ and $j = 1,\ldots, n$. The reference ideal is given by the identity vector (1,1,...,1). Accordingly, the weighted reference ideal equals the vector of weights, w.

Step 6. Calculate the relative index of each alternative throughout the following expression:

$$R_i = \frac{I_i^-}{I_i^+ + I_i^-}, i = 1, \ldots, m, \text{ where } R_i \in (0, 1)\,. \quad (3)$$

Step 7. Sort all the alternatives into descending order according to their relative indexes. Thus, the alternatives appearing at the top become the best solutions, i.e., the ones closest to the reference ideal alternative.

2.3. Fuzzy sets

Due to vagueness or uncertainty, it is sometimes difficult to quantitatively express concepts as diverse as the maturity of a technology, the comfort of a seat or even the maneuverability of a system. Under such circumstances, the use of linguistic labels linked with fuzzy numbers is an excellent way of quantifying criteria or factors that at first sight are qualitative in nature. The fuzzy set theory [38,39] deals with this type of uncertainty and has enabled a large number of complex real problems to be solved [15,40,41]. These problems frequently require the management of situations in which the pertinent data and the sequences of potential actions are unknown. When a decision maker has to tackle a problem of ranking m alternatives $A_1, A_2, \ldots, A_m$ with respect to n criteria $C_1, C_2, \ldots, C_n$ then the difficulty of assigning numbers to alternatives in terms of these criteria is presented. Thus, crisp MCDM methods may not be directly applicable in a fuzzy environment. Henceforth, fuzzy alternatives and decision criteria will be denoted as $\widetilde{A}_i$ and $\widetilde{C}_i$, respectively, in order to distinguish them from their crisp version counterparts.

Due to their simplicity, triangular fuzzy numbers defined through triangular membership functions are used in the fuzzy version of the RIM approach. Triangular fuzzy numbers, its theory and basic operations are described in detail in [11,42].

2.4. The Fuzzy Reference Ideal Method (FRIM)

The Fuzzy RIM (FRIM) approach [22] has been developed to work on sets of fuzzy numbers rather than on crisp numbers. In that case, the distance between two fuzzy numbers $\left(\tilde{X}_{ij}, \widetilde{D}_{ij}\right)$ is calculated via expression (4):

$$dist\left(\tilde{X}_{ij}, \widetilde{D}_{ij}\right) = \sqrt{\frac{1}{3}\left((x_1 - d_1)^2 + (x_2 - d_2)^2 + (x_3 - d_3)^2\right)} \quad (4)$$

Likewise, the minimal distance to a fuzzy interval must be reformulated (expression (5)):

$$d_{min}^*\left(\tilde{X}_{ij}, \left[I\tilde{R}_j\right]\right) = min\left(dist\left(\tilde{X}_{ij}, \tilde{C}_j\right), dist\left(\tilde{X}_{ij}, \widetilde{D}_j\right)\right) \quad (5)$$

where $dist\left(\tilde{X}_{ij}, \tilde{C}_j\right)$ and $dist\left(\tilde{X}_{ij}, \widetilde{D}_j\right)$ are obtained through expression 4. Observe that $\tilde{X}$, $\tilde{C}$ and $\tilde{D}$ are triangular fuzzy numbers and $I\tilde{R}_j = \left[\tilde{C}_j, \widetilde{D}_j\right]$ represents the interval of the Reference Ideal.

When it is necessary to operate with fuzzy numbers, the normalization process of the decision matrix defined by expression 1 should not be applied. Therefore, that normalization process must be reformulated as the following (expression (6)):

$$f^*\left(\tilde{X}_{ij}, \left[\tilde{R}_j\right], \left[I\tilde{R}_j\right]\right) = \begin{cases} 1 & if \quad \tilde{X}_{ij} \in \left[I\tilde{R}_j\right] \\ 1 - \dfrac{d_{min}^*\left(\tilde{X}_{ij}, \left[I\tilde{R}_j\right]\right)}{dist\left(\tilde{A}_j, \tilde{C}_j\right)} & if \ \tilde{X}_{ij} \in \left[\tilde{A}_j, \tilde{C}_j\right] \wedge \tilde{X}_{ij} \notin \left[I\tilde{R}_j\right] \\ & \wedge\, dist\left(\tilde{A}_j, \tilde{C}_j\right) \neq 0 \\ 1 - \dfrac{d_{min}^*\left(\tilde{X}_{ij}, \left[I\tilde{R}_j\right]\right)}{dist\left(\widetilde{D}_j, \tilde{B}_j\right)} & if \ \tilde{X}_{ij} \in \left[\widetilde{D}_j, \tilde{B}_j\right] \wedge \tilde{X}_{ij} \notin \left[I\tilde{R}_j\right] \\ & \wedge\, dist\left(\widetilde{D}_j, \tilde{B}_j\right) \neq 0 \\ 0 & in\ other\ case \end{cases} \quad (6)$$

where $\tilde{R}_j = \left[\tilde{A}_j, \tilde{B}_j\right]$ and $I\tilde{R}_j = \left[\tilde{C}_j, \widetilde{D}_j\right]$ represent the range and the Reference Ideal interval, respectively.

Once we have obtained the normalization function for each value of the decision matrix, we define the normalized decision matrix, and the rest of the steps of the RIM algorithm (from step 4 to step 7) can be carried out.

The phase of alternatives assessment of this work is carried out through the RIM and FRIM approaches. This assessment enables a ranking of alternatives based on an ideal reference alternative to be obtained.

3. The decision problem: Selection of military advanced training aircraft for the Spanish Air Force

The 23rd Fighter and Attack Training Wing, located in Talavera la Real (Spain), is the Jets School of the Spanish Air Force. The main task of this school consists in training young officers who, once they have completed their advanced training phase, will be ready to fly fighters of 4th plus generation such as the EF-18 Hornet or the Eurofighter TYPHOON. This training course currently uses the Northrop F-5 Freedom Fighter. Although this training aircraft was introduced in the early 1970s, it has been subjected to several updates which have turned it into an exceptional trainer. However, it is falling behind the times in terms of facing new requirements and training standards. Due to that fact, in the short term, it will be necessary to pose the following questions: Could new updates be made on the current trainer? or would it be more appropriate to find another advanced trainer to replace it? In order to deal with this decision problem, it is not only necessary to take into consideration the candidate aircraft which could replace the current trainer, but also to analyze the requirements (criteria) that such candidates must fulfill in depth. Therefore, the Logistics Support Command (MALOG) and the Air Staff (EMA) of the Spanish Air Force are considering several candidates [43].

3.1. Alternatives definition

The alternatives evaluated in this case study correspond to fourth generation aircraft which, due to their performance and features, have stood out in their field in the last decade. Some of the main technological features of this generation of aircraft include Pulse-doppler radar; high maneuverability; look-down/ shoot-down missiles, etc. [44]. These advanced military aircraft are the options which are closest to the current advanced trainer in the 23rd Fighter and Attack Training Wing of the Spanish Air Force.

The first two proposed alternatives (KAI-T-50 Golden Eagle and Alenia Aermacchi M-346 Master) are being considered by the decision-makers (EMA and MALOG). The third candidate, Yakovlev YAK-130, has been included in this study as a result of having similar features to alternatives A_1 and A_2. The current trainer (Northrop F-5 Freedom Fighter) is also taken into consideration with the aim of analyzing if a complete modernization program could be justified. Finally, it is important to highlight that the SPAF is weighing up another trainer, the Boeing-Saab T-X (an American/Swedish advanced jet trainer). However, the present work has not considered it as it is still being developed [45].

Alternative A_1.- KAI-T-50 Golden Eagle. Advanced trainer jet jointly developed by Korean Aerospace Industries (KAI) and Lockheed Martin, manufactured in the late 1990s. It is powered by a General Electric F404-GE-402 turbofan [46]. Because of its distinguished aerodynamic performance, it has proved to have excellent maneuverability and flying qualities [47]. It has advanced radar, sensors, weapon and digital avionics which enable future upgrades that will allow the trainer to closely mimic the fourth-generation fighter [48]. Its digital fly-by-wire (FBW) flight control system provides carefree handling, high thrust-to-weight ratio, high sustained g and high angle-of-attack (AoA) capabilities. In fact, its maneuverability and advanced systems have been designed to prepare future pilots to fly next generation fighters such as the Eurofighter Typhoon, Dassault Rafale and Lockheed Martin F-35. This military aircraft has been in service in the Republic of Korea Air Force (ROKAF) since 2005.

Alternative A_2.- Alenia Aermacchi M-346 Master. This Italian-enterprise trainer started out as a joint project between Italy and Russia through Alenia Aermacchi and Yakovlev Design Bureau manufacturers, respectively [49]. Its power plant consists of two Honeywell/ITEC F124-GA-200 turbofan engines. Similarly to the T-50 Golden Eagle, its aerodynamic design and a FBW flight control system provide maneuverability and controllability at a very high angle-of-attack. Its cockpit is representative of the latest-generation combat aircraft with an MK16 seat; the most relevant operational equipment is located in front of the pilot. It also has three multifunction displays (MFD), and the rear cockpit instructor, thanks to the high position and an own Head-Up Display (HUD), scarcely loses visibility and so provides the ability to utilize the FBW control system in various training modes [50,51]. Nowadays, this aircraft is in service in the Italian Air Force, the Israeli Air Force and the Republic of Singapore Air Force, with the Polish Ministry of Defence having recently signed contracts to acquire eight of these aircraft; deliveries are expected to conclude by 2022.

Alternative A_3.- Yakovlev YAK-130. This two-seat advanced jet trainer and light fighter entered service at the military pilot training academy of the Russian Air Force in 2009. It was originally fruit of the common project named Yak /AEM-130 between Yakovlev Design Bureau and Alenia Aermacchi manufacturers which, following their split, each developed an advanced military trainer (YAK-130 and M-346, respectively). The YAK-130 includes two AI-222-25 turbofan engines [49]. Its high number of wing suspension points provides a combat load capability of up to 3000 kg [52]. This trainer also has the ability to operate in a wide range of angles of attack [53]. With respect to the cockpit, it is also equipped with FBW flight control system, three MFDs, zero–zero ejection seats and a navigation suite with laser gyroscopes and GLONASS/NAVSTAR global positioning. It is very resistant to going into a spin and, due to preventing the entry of FOD (Foreign Object Damage), it is certified to take off and land on unpaved runways. The YAK-130 has received a very positive market response, with 140 units already operative worldwide. Its first users were the Russian Air Force, the Algerian Air Force, the Bangladesh Air Force and the Belarusian Air Force.

Alternative A_4.- Northrop F-5 Freedom Fighter (current trainer in the Spanish Air Force). The Northrop F-5 Freedom Fighter is also included as an alternative to carry out a comparison process with the rest of the proposed candidates. This supersonic light fighter was introduced by the Northrop Corporation manufacturer in the early 1960s. It was postulated as an inexpensive alternative to the F-111 in the Vietnam War [54,55]. It is powered by two General Electric J85-GE-21 afterburning turbojet engines. More than 2000 units have been built since its entry into service and its first users were the United States Navy, the Republic of China Air Force, the Republic of Korea Air Force, the Islamic Republic of Iran Air Force, the Brazilian Air Force, to mention just a few of them. The 23rd Fighter and Attack Training Wing (Talavera la Real, Spain) currently has 19 operative units of this trainer and light fighter. The inclusion of the current trainer will offer the chance to observe its position in the comparison process. In this way, an exhaustive modernization program which was able to fulfill the new requirements and training standards would be analyzed.

3.2. Criteria definition

A group of experts consisting of 10 pilot trainers of the 23rd Fighter and Attack Training Wing took part in this step of the study. They established the main criteria which should be taken into consideration. Thanks to their experience as instructors and trained pilots, not only was it possible to define the most important technical parameters for the aeronautical training [56,57], but also to detect the existence of other qualitative factors, such as maneuverability or ergonomic conditions, which should be included. In fact, due to the similarity of technical features in most military aircraft, qualitative and subjective criteria are becoming key factors [19].

Criterion C_2.- Combat ceiling (ft). The maximum altitude at which an aircraft can continue to perform maneuvers effectively. Due to the density and speed of the air, and the variation of the lift in its wings, as the aircraft gains altitude, it loses maneuverability. For all this, this criterion is a relevant factor in combat sceneries since it defines the limit from which an aircraft would be at an inferiority level. This criterion has a quantitative nature and, according to the group of experts, its ideal value should be between 43 000 and 45 000 ft.

Criterion C_2.- Endurance (hours). The maximum time that an airplane stays in the air on a tank of fuel. This criterion is relevant since the duration of the flight classes and the operating capacity of the missions depend on it. This criterion is also quantitative, and its ideal value should be between three and four hours.

Criterion C_3.- Thrust (kN). The force which moves an aircraft through the air. This factor provides information about how an aircraft can react in flight in different situations. According to the experts, this quantitative criterion should be between 50 and 55 kN.

Criterion C_4.- Weight at take-off (lb). The maximum weight at which an aircraft is authorized to take off. When it comes to carrying out a mission, this factor is relevant since the load can be both armament and fuel. It also provides information about the characteristics of the aircraft at take-off and how its capacity to react to a problem. The ideal values of this quantitative criterion should be between 21 000 and 22 000 lb.

Table 2
Order of importance of criteria for each expert.

E_1	$C_{12} > C_9 > C_{10} > C_3 > C_1 > C_{11} > C_{13} > C_2 > C_5 > C_8 > C_6 > C_4 > C_7$
E_2	$C_{12} > C_9 > C_{10} > C_{13} > C_{11} > C_1 > C_3 > C_2 > C_5 > C_8 > C_6 > C_7 > C_4$
E_3	$C_{12} > C_9 > C_{10} > C_{13} > C_{11} > C_1 > C_3 > C_2 > C_5 > C_8 > C_6 > C_4 > C_7$
E_4	$C_{12} > C_9 > C_{13} > C_{10} > C_{11} > C_1 > C_3 > C_2 > C_5 > C_8 > C_6 > C_4 > C_7$
E_5	$C_{12} > C_9 > C_{13} > C_{10} > C_{11} > C_1 > C_2 > C_3 > C_5 > C_8 > C_6 > C_4 > C_7$
E_6	$C_{12} > C_9 > C_{10} > C_{13} > C_{11} > C_1 > C_2 > C_3 > C_5 > C_8 > C_6 > C_4 > C_7$
E_7	$C_{12} > C_9 > C_{10} > C_{11} > C_1 > C_{13} > C_3 > C_2 > C_5 > C_8 > C_6 > C_4 > C_7$

Criterion C_5.- Operational speed (kt). The maximum speed at which an aircraft can perform a given mission without jeopardizing its integrity. This factor has an influence in the types of training mission. Interception missions demand high speeds while air-to-surface combat missions do not require such high speeds. This criterion also has a quantitative nature and, according to the group of experts, its ideal value should be between 800 and 820 kt.

Criterion C_6.- Take-off race (ft). The distance along the ground required by an aircraft, starting from the releasing of brakes, to lift the wheels off the ground. This factor varies depending on the different configurations of the training aircraft and is an important parameter in take-off emergencies. The lower the take-off race, the greater the distance of remaining track there will be in the event of failure. An ideal value of 500 ft is considered, with a minimum value being desirable.

Criterion C_7.- Rotational speed (kt). Velocity below the take-off speed in which an aircraft has enough lift to start performing depth movements in the flight controls, without increasing the drag and take-off distance. This factor provides information about the flight envelope of the aircraft. This quantitative criterion should be between 113 and 120 kt.

Criterion C_8.- Range (nm). The maximum distance that an aircraft stays in the air with the maximum fuel and minimum weight, in a straight line and without returning to the starting point. This enables the different training missions to be designed. For example, this factor provides the radius of influence of the aircraft in combat missions. According to the experts, this quantitative criterion should be between 1000 and 1020 nm.

Criterion C_9.- Tactical capability (qualitative). This criterion combines parameters such as the load capacity of the aircraft, the different types of armament, the avionics systems and the different simulation programs, which provide tactical advantages for each alternative. Therefore, this factor not only enables to design different training missions, but also plays a relevant role in the familiarization with new weapons systems and simulation programs.

Criterion C_{10}.- Maneuverability (qualitative). The ability of movement and agility of an aircraft. It is based on factors such as positive and negative limit load factors, cruising speed and service ceiling which are able to generate an own surround flight.

Criterion C_{11}.- Ergonomics (qualitative). The suitable space, ease of access to the main commands, comfort in the execution of procedures and the advantages which the position of the different controls in the cockpit gives the trainee officer and instructor. From the point of view of training missions, this criterion allows not only for familiarizing the trainee with the commands and control, but also for facilitating the visibility and the control of the instructor from the rear cockpit.

Criterion C_{12}.- Compatibility (qualitative). The degree of agreement with the infrastructures and procedures that the Spanish Air Force has in their different units and air bases. This criterion involves diverse factors such as the know-how of the platform and its compatibility with other settled aircraft with the aim of obtaining spare parts and carrying out the maintenance service.

Criterion C_{13}.- Cost (qualitative). Although the unit cost of a specific aircraft has a quantitative nature, in the acquisition process of this type of platforms, its value depends on different factors of a qualitative nature such as business strategies across countries, politics and diplomatic relations, etc. Due to that fact, this criterion presents a major subjective character.

3.3. Problem structure

To assess the alternatives (military advanced training aircraft) based on the criteria mentioned above, a prior stage has to be performed; obtaining the weights of coefficient of importance of the criteria. A group of experts (seven pilot trainers from the 23rd Fighter and Attack Training Wing) performed this extraction of knowledge by filling out a survey based on the application of the AHP methodology.

3.3.1. Determination of the weight of the criteria

To determine the weights of the criteria, a questionnaire based on the AHP methodology and composing three questions is carried out:

Question 1: Do you believe that the thirteen criteria considered have the same weight?

If the answer was yes, then $w_i = w_j = 1/n$ for all i, j. Thus, the weights of the criteria have already been defined since these will all have the same value. Otherwise, i.e., if the answer was no (not all the criteria have equal importance), then the next question of the questionnaire should be posed.

Question 2: List the criteria in descending importance

This step provides information about the order of importance of each of the criteria. In that case, the group of experts has considered that certain criteria should have a greater weight than others. The order of importance of these criteria is shown in Table 2.

Once the group of experts has provided the order of importance of the criteria, it is necessary to establish a comparison process between them. To do that, a third and final question should be addressed:

Question 3: Compare the criterion to be considered first with respect to that considered secondly and successively, using the following labels {(EI), (WMI), (SMI), (VSMI), (AMI)} which correspond to Saaty's scale of valuation in the pair-wise comparison process (see** Section 2.1**)

As an example of such comparison process, the judgments provided by Expert 1 (E_1) are shown in Table 3:

The information detailed above for E_1 (Table 3) would also be carried out for the rest of the experts. Considering that all experts are equally important in this study case, a homogeneous aggregation of that information via arithmetic average is made. Therefore, the weights of each criterion (Table 4 and Fig. 2) are obtained:

Homogeneous aggregation shows the qualitative criteria to be more important than the quantitative criteria (Fig. 2). The most important criterion is C_{12} (Compatibility), while the second most important is C_9 (Tactical capability). The least important criteria are C_6 (Take-off race), C_4 (Weight at take-off) and C_7 (Rotational speed). In order to verify the judgments provided by the experts through the AHP methodology, the consistency ratio (CR) is calculated. Because the value of this ratio is less than 0.1 for each one of the experts, their judgments need not be revised.

Table 3
Matrix C of judgments provided by E_1.

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}	C_{12}	C_{13}
C_1	1	1	1/3	3	1	5	5	1	3	3	1/3	1/5	1/3
C_2	1	1	1	5	3	3	5	3	1/3	1/3	1	1/3	1
C_3	3	1	1	5	1	3	7	1	1	1	1	1	1
C_4	1/3	1/5	1/5	1	1/5	1	1	1/3	1/7	1/7	1/5	1/9	1/5
C_5	1	1/3	1	5	1	3	3	1	1/7	1	1	1/3	1
C_6	1/5	1/3	1/3	1	1/3	1	1	1/3	1/5	1/7	1/5	1/7	1/5
C_7	1/5	1/5	1/7	1	1/3	1	1	1/5	1/7	1/7	1/5	1/7	1/7
C_8	1	1/3	1	3	1	3	5	1	1/7	1/3	1	1/5	1
C_9	1/3	3	1	7	7	5	7	7	1	1	1	1	1
C_{10}	1/3	3	1	7	1	7	7	3	1	1	1	1	3
C_{11}	3	1	1	5	1	5	5	1	1	1	1	1/3	1
C_{12}	5	3	1	9	3	7	7	5	1	1	3	1	3
C_{13}	3	1	1	5	1	5	7	1	1	1/3	1	1/3	1

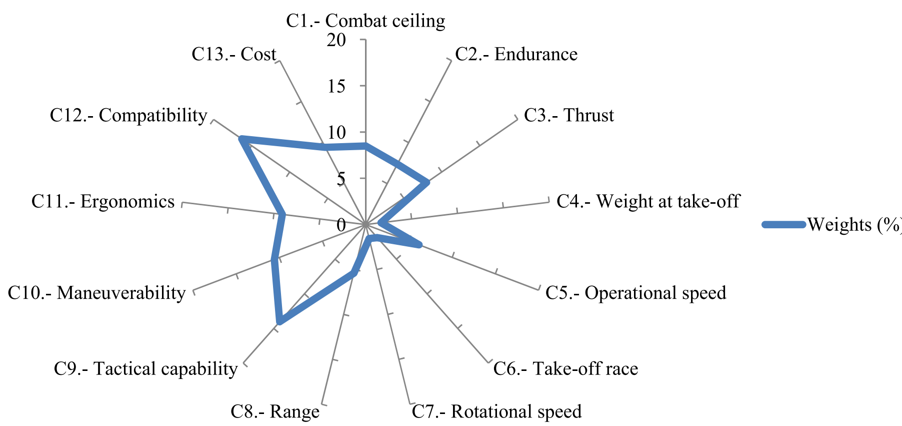

Fig. 2. Graphical representation of weights. The figure illustrates the importance of the qualitative criteria (C_9, C_{10}, C_{11}, C_{12} and C_{13}), which are located on the left side of the figure, to the detriment of the quantitative criteria.

Table 4
Weights of criteria through experts' homogeneous aggregation.

Criteria	Weights (%)
C_1.- Combat ceiling	8.49
C_2.- Endurance	7.37
C_3.- Thrust	8.00
C_4.- Weight at take-off	1.67
C_5.- Operational speed	6.18
C_6.- Take-off race	1.89
C_7.- Rotational speed	1.56
C_8.- Range	5.42
C_9.- Tactical capability	14.03
C_{10}.- Maneuverability	10.57
C_{11}.- Ergonomics	9.09
C_{12}.- Compatibility	16.30
C_{13}.- Cost	9.43

3.3.2. Obtaining assessments of the alternatives

After obtaining the criteria weights, the alternatives for each of the criteria must be evaluated based on a reference ideal alternative through the RIM and FRIM approaches, due to the co-existence of quantitative and qualitative criteria. Firstly, the RIM methodology is applied until step 3 (obtaining the normalized matrix) based on quantitative criteria (from criterion C_2 to Criterion C_8). Due to that fact, this first process is named a quantitative process. Subsequently, the qualitative criteria (from criterion C_9 to Criterion C_{13}) are taken into account in the assessment process through the FRIM approach. Again, this second process (named a qualitative process) ranges until step 3 of this methodology (obtaining the normalized matrix). Once these normalized matrices have been obtained, both matrices are unified into a single normalized matrix and the remaining steps of the RIM approach are applied with the aim of obtaining a ranking of alternatives.

(A) Quantitative Process.

Firstly, the decision matrix of alternatives and criteria must be created. In order to do so, a numerical value for each criterion and alternative must be associated. These values (Table 5) have been numerically quantified from literature data [1,50,52,58,59] and the trained test pilots' report [43].

In order to apply the RIM approach, not only should a Reference Ideal alternative be established, but also the Range that belongs to the universe of discourse of the decision problem must be defined. The group of experts composing 10 pilot trainers from the 23rd Fighter and Attack Training Wing has determined such concepts (Reference Ideal and Range) for this specific study case (Table 6).

Once the work context (Table 6) and the decision matrix (Table 5) have been defined, step 3 of the RIM approach is carried out with the aim of obtaining the normalized matrix (Table 7) through expressions (1) and (2).

(B) Qualitative Process.

A similar process to the quantitative process is carried out in this step. However, and due to the existence of qualitative criteria, the fuzzy version of RIM approach (FRIM) must be applied. To do so, the group of experts once again intervenes to provide the valuation matrix of alternatives and qualitative criteria (from criterion C_9 to criterion C_{13}). Therefore, they must define not only a range for these criteria and a Reference Ideal alternative (Table 8), but also carry out a qualitative assessment based on two linguistic labels, L_1 and L_2.

The linguistic labels used to describe the four alternatives for each of the criteria, their qualitative assessment, and their fuzzy

Table 5
Decision matrix of the quantitative process.

	C_1 (ft)	C_2 (h)	C_3 (kN)	C_4 (lb)	C_5 (kt)	C_6 (ft)	C_7 (ft)	C_8 (nm)
A_1	47 999	2	53	26 422	860	600	124	999
A_2	44 997	4	55.6	21 165	788	500	115	1021
A_3	42 651	3	49	19 836	572	500	113	1375
A_4	55 118	2	31.2	24 715	1012	1000	149	759

Table 6
Values of the Range and Reference Ideal in the quantitative process.

Range	C_1 (ft)	C_2 (h)	C_3 (kN)	C_4 (lb)	C_5 (kt)	C_6 (ft)	C_7 (ft)	C_8 (nm)
A	42 000	1	30	19 000	500	0	100	750
B	55 200	5	56	27 000	1020	1020	150	1400
Reference Ideal	C_1 (ft)	C_2 (h)	C_3 (kN)	C_4 (lb)	C_5 (kt)	C_6 (ft)	C_7 (ft)	C_8 (nm)
C	43 000	3	50	21 000	800	0	113	1000
D	45 000	4	55	22 000	820	500	120	1020

Table 7
Normalized matrix in the quantitative process.

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
A_1	0.706	0.500	1.000	0.116	0.800	0.808	0.867	1.000
A_2	1.000	1.000	0.400	1.000	0.960	1.000	1.000	0.997
A_3	0.651	1.000	0.950	0.418	0.240	1.000	1.000	0.066
A_4	0.008	0.500	0.060	0.457	0.040	0.038	0.033	0.036

Table 8
Values of the Range and Reference Ideal in the qualitative process.

Range	C_9	C_{10}	C_{11}	C_{12}	C_{13}
A	[0, 1, 2]	[0, 1, 2]	[0, 1, 2]	[0, 1, 2]	[0, 1, 2]
B	[8, 9, 10]	[8, 9, 10]	[8, 9, 10]	[8, 9, 10]	[8, 9, 10]
Reference Ideal	C_9	C_{10}	C_{11}	C_{12}	C_{13}
C	[5, 7, 9]	[5, 7, 9]	[5, 7, 9]	[5, 7, 9]	[3, 5, 7]
D	[7, 9, 10]	[7, 9, 10]	[5, 7, 9]	[7, 9, 10]	[7, 9, 10]

Table 9
Linguistic label type.

Linguistic label L_1	Linguistic label L_2	Fuzzy numbers
Very Bad VB	Very Low VL	[0, 1, 3]
Bad B	Low L	[1, 3, 5]
Medium M	Medium M	[3, 5, 7]
Good G	High H	[5, 7, 9]
Very Good VG	Very High VH	[7, 9, 10]

numbers are shown in Table 9. Regarding criteria C_9, C_{10}, C_{11}, and C_{12} the different trainers are assessed using type L_1 linguistic labels. For criterion C_{13}, different alternatives are assessed through type L_2 linguistic labels.

Table 10 shows the arithmetic average of all the qualitative criteria assessments provided by the experts for each of the alternatives.

Applying step 3 of the FRIM approach through the information provided in Tables 8 and 10, the normalized decision matrix is obtained (Table 11) via expressions (4), (5) and (6).

(C) Unified Process.

The quantitative and qualitative processes are combined in a unique process in which the normalized matrices (Tables 7 and 11) can be unified generating a single matrix of four alternatives and 13 criteria with crisp or real values. Hereupon, the weighted normalized matrix of these alternatives and criteria is calculated through the weights of the criteria (previously obtained in Section 3.3.1). This corresponds with the following step of the RIM approach (step 4). Likewise, the rest of the steps of the RIM algorithm (from step 5 to step 7) can be carried out, concluding with the obtaining of a ranking of alternatives based on the ideal reference alternative.

3.4. Results

After obtaining the weighted normalized matrix (step 4) and calculating the variation to the normalized reference ideal ($I_i{}^+$ and $I_i{}^-$) and the relative index (R_i) for each alternative A_i (steps 5 and 6), a ranking of alternatives based on the reference ideal alternative is generated (Table 12).

According to Table 12, the candidate aircraft located in 1st position is alternative A_2 (Alenia Aermacchi M-346 Master). The second alternative, very close to the first option, is alternative A_1 (KAI-T-50 Golden Eagle). The values obtained for the third and fourth candidates are similar.

3.5. Discussion

This ranking was shown to the group of experts composed of pilot trainers from the 23rd Fighter and Attack Training Wing. They confirmed such results as being in line with their judgments. Fig. 3, which shows a graphical representation of the normalized decision matrix, can explain these results. Alternative A_2 has the best score in almost all the criteria with the exception of criteria C_3 (Thrust), C_8 (Range), and C_{11} (Ergonomics), where alternative A_1 obtains the best scores.

The first two alternatives according to the RIM ranking (A_1 and A_2) coincide with the most logical options which are being considered by the Spanish Ministry of Defence. In addition, both the Italian manufacturer of the Alenia Aermacchi (Leonardo) and its Korean counterpart KAI (manufacturer of the T-50 Golden Eagle) have recently proposed acquisition programs for their aircraft to the Spanish Air Force [60].

Furthermore, according to the group of experts, the alternative positioned first (Alenia Aermacchi M-346 Master) highlights both the modular architecture of its digital avionics system and its ease of upgrade to future needs as an attack aircraft; this advanced training aircraft is prepared to carry both air-to-air and air-to-surface weapons [61]. This characteristic makes it an excellent fighter trainer for future Eurofighter TYPHOON pilots and even for 5th generation fighter pilots. In this way, the cost of these advanced training phases could be lowered by concentrating them on this model.

With regard to the rest of alternatives (A_3 and A_4), the experts considered to highlight the positions of the alternatives A_3 and A_4 since the third position corresponds to alternative A_4 (Northrop F-5 Freedom Fighter), the current trainer aircraft, beating alternative A_3 (Yakovlev YAK-130). The justification for the exchange of positions between alternatives A_3 and A_4 is explained by the greater weight of the qualitative criteria to the detriment of the quantitative criteria. Due to that fact, although alternative A_3 has better scores than alternative A_4 in most quantitative criteria, said

Table 10
Decision matrix of the qualitative process.

	C_9			C_{10}			C_{11}			C_{12}			C_{13}		
A_1	[6.6,	8.6,	9.8]	[6.6,	8.6,	9.8]	[5.0,	7.0,	8.8]	[4.2,	6.2,	8.0]	[0.7,	2.4,	4.4]
A_2	[6.8,	8.8,	9.9]	[6.8,	8.8,	9.9]	[5.2,	7.2,	9.0]	[6.2,	8.2,	9.6]	[3.6,	5.6,	7.6]
A_3	[6.8,	8.8,	9.9]	[5.6,	7.6,	9.3]	[4.0,	6.0,	8.0]	[0.4,	1.8,	3.8]	[6.8,	8.8,	9.9]
A_4	[4.0,	6.0,	8.0]	[2.4,	4.4,	6.4]	[3.6,	5.6,	7.6]	[7.0,	9.0,	10]	[7.0,	9.0,	10]

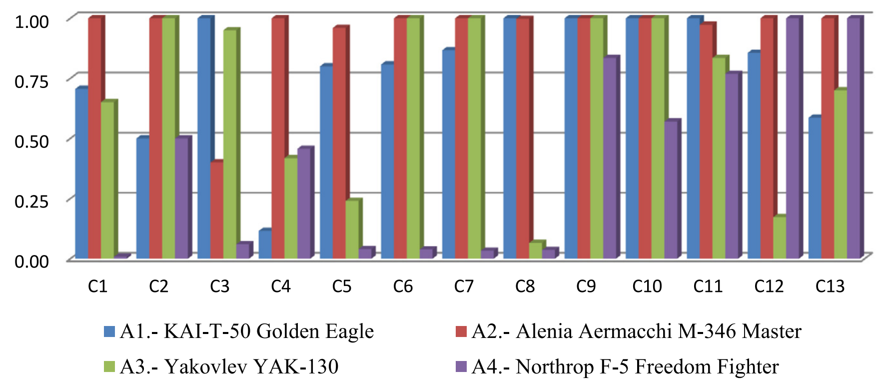

Fig. 3. Graphical representation of the normalized decision matrix. The figure illustrates the normalized values of the criteria for each of the alternatives to be evaluated.

alternative (A_3) is relegated to the fourth position. This result demonstrates the confidence that experts have in the current advanced training aircraft (Northrop F-5 Freedom Fighter), opening the door to undertaking a complete modernization program. The experts indicated that in such a case, a thorough and exhaustive technical and economic viability study should be carried out.

Furthermore, as a result of the fact that the A3 alternative (Yakovlev YAK-130) has a very low value in the Compatibility criterion (C_{12}), which is also the criterion with the greatest weight, its overall value decreases. With regard to this alternative, the experts argued that since it is an aircraft manufactured by a nation that does not belong to NATO, and although in general it presents very good technical characteristics, its compatibility with other aircraft or platforms is a considerable limitation.

3.5.1. Limitations of the study

This study presents two important drawbacks; the first one consists of the application of the AHP methodology when the number of criteria increases considerably. The author of AHP, Thomas Saaty, recommends not to compare more than 7 ± 2 elements in the same level of its hierarchy with the aim of carrying out the reciprocal comparisons [62,63]. The group of experts selected the criteria involved in this specific study case and their number (13 criteria) is higher than the range defined by Saaty. That fact could have caused the value of the consistency ratio (*CR*) to be greater than 0.1 for some experts and their judgments should be revised. In such case, the problem could be tackled with the creation of two levels in the hierarchy structure (criteria and sub-criteria). In this way, the number of reciprocal comparisons between elements on the same level would decrease.

If the previous option is not possible i.e., all the criteria are located on the same level of the hierarchy, we can resort to Question 2 of the survey (see Section 3.3.1). In this way, a process that only includes the comparison of the criterion or criteria considered in the first place with respect to the rest could be carried out. A potential inconsistency may be avoided with this procedure, especially when a survey involves many criteria. This process constitutes an approach to the standard AHP methodology and has been used previously in the scientific literature [19,64–66].

The second limitation is not a disadvantage per se, but is a relevant factor to take into account. It is the range of the ideal interval for each criterion in the stage of evaluating the alternatives with the RIM methodology. Assigning the values by the group of experts is no easy task because if the intervals are narrow then the solution will be more precise and delimited. Due to that fact, the group of experts mentioned that such ranges should be as specific and restrictive as the information we have about that criterion.

Table 11
Normalized matrix in the qualitative process.

	C_9	C_{10}	C_{11}	C_{12}	C_{13}
A_1	1.000	1.000	1.000	0.856	0.586
A_2	1.000	1.000	0.973	1.000	1.000
A_3	1.000	1.000	0.835	0.173	0.700
A_4	0.835	0.571	0.769	1.000	1.000

Table 12
The ranking obtained using the Fuzzy RIM method.

Alternatives	I_i^+	I_i^-	R_i	Ranking
A_1.- KAI-T-50 Golden Eagle	0.067	0.281	0.808	2
A_2.- Alenia Aermacchi M-346 Master	0.048	0.308	0.865	1
A_3.- Yakovlev YAK-130	0.158	0.239	0.602	4
A_4.- Northrop F-5 Freedom Fighter	0.155	0.243	0.611	3

4. Sensitivity analysis

A sensitivity analysis was carried out in order to validate the robustness and strength of the results obtained above. Two new considerations were therefore made: firstly, the four proposed alternatives were re-analyzed using the RIM and FRIM approaches but with the assumption that all the criteria had the same weight. Secondly, the alternatives were analyzed with some MCDM methods with the aim of comparing the order of priority of these alternatives.

4.1. Variation of the weights

The results obtained by applying the RIM and FRIM methodologies for the four alternatives assuming that all the criteria have the same importance are displayed in Table 13.

Table 13
Comparison of the alternatives based on the weights of the criteria.

Alternatives	Weights through experts group (Table 4)		All the criteria with the same weight	
	R_i	Ranking	R_i	Ranking
A_1.- KAI-T-50 Golden Eagle	0.808	2	0.715	2
A_2.- Alenia Aermacchi M-346 Master	0.865	1	0.852	1
A_3.- Yakovlev YAK-130	0.602	4	0.629	3
A_4.- Northrop F-5 Freedom Fighter	0.611	3	0.444	4

The obtained results shown in Table 13 indicate that although the position of the first two alternatives (A_1 and A_2) has not changed, the difference between both has increased slightly. However, this is not the case with alternatives A_3 and A_4, which have exchanged their positions. The explanation for that fact lies in the values of the quantitative criteria of alternative A_3, which are, with the exception of criterion C_4 (weight at take-off), higher than those of alternative A_4.

Consequently, such a sensitivity analysis allows us to discard a bias underlying the expert judgments. Hence, these results suggest that the judgments provided by the experts via AHP-based questionnaires have not influenced the two first alternatives.

4.2. Comparison with other MCDM methods

In order to carry out a sensitivity analysis according to different points of view and hence, to provide consistency to the obtained results, a comparative study among MCDM methods is performed. The priority orders obtained with the RIM and FRIM approaches are compared with the following MCDM methods: the TOPSIS method [13], the Weighted Sum Model (WSM) method [40], as well as the revised AHP methodology [62] i.e., the "ideal mode" AHP [35] to check if a ranking inconsistency can occur.

With regard to the application of the TOPSIS method to this specific case study it is necessary to make some clarifications beforehand; the TOPSIS method is based on the concept of positive ideal solution and negative ideal solution where the criteria which influence the decision problem are criteria to maximize or minimize. Although that does not correspond with the study case proposed in this work, it is possible to assimilate the criteria that could tend to be maximized as benefit criteria. Likewise, criteria which could tend to be minimized can be defined as cost criteria. In this way, criteria C_2 (Combat ceiling), C_6 (Take-off race), C_7 (Rotational speed), and C_{13} (Cost) would be defined as criteria to minimize, while the rest of the criteria (C_2.-Endurance, C_3.-Thrust, C_4.-Weight at take-off, C_5.-Operational speed, C_8.-Range, C_9.-Tactical capability, C_{10}.-Maneuverability, C_{11}.-Ergonomics, and C_{12}.-Compatibility) would be criteria to maximize.

We apply the revised AHP methodology assuming that the entry x_{ij} in the $m\ x\ n$ valuation matrix represents the relative value of alternative A_i and corresponds to the values of the normalized matrix for all the alternatives and criteria.

Therefore, on the basis of joining the normalized matrices (Tables 7 and 11) to generate a single normalized matrix for all the alternatives and criteria, it is possible to apply the TOPSIS algorithm and the rest of the MCDM approaches mentioned above (WSM and revised AHP) to obtain their respective rankings taking into account the weights of the criteria provided by the group of experts (Table 14).

From the results shown in Table 14, it must be highlighted that the order of the first two alternatives changed upon applying the TOPSIS method. The best alternative corresponded to A_1.- KAI-T-50 Golden Eagle rather than alternative A_2.- Alenia Aermacchi M-346 Master, which was the first one according to the FRIM and RIM approaches.

We can also see in said table that the best alternative obtained by the FRIM methodology (alternative A_2.- Alenia Aermacchi M-346 Master) was the same as that obtained by the WSM and Revised AHP methodologies. However, the subsequent alternatives (alternatives A_3.-Yakovlev YAK-130 and A_4.-Northrop F-5 Freedom Fighter) have swopped positions.

These facts confirm the validity of the FRIM and RIM approaches for solving this type of decision problem. These methodologies (RIM and FRIM) enable alternatives to be evaluated without the need for the ideal values of the criteria to be maximums or minimums; these values can even belong to an interval.

5. Conclusions

This paper shows the possibility of combining classical Multi-Criteria Decision Making (MCDM) methods such as the AHP methodology with the fuzzy version of a recent MCDM method, the RIM approach, to determine the best advanced trainer aircraft among a set of alternatives for the Jets School of the Spanish Air Force. Firstly, the AHP methodology allows us to determine the criteria weights, based on the knowledge of a group of experts composed of pilot trainers from the 23rd Fighter and Attack Training Wing. The most relevant criteria turned out to be criteria of a qualitative nature, the most prominent being Compatibility (C_{12}), Tactical capability (C_9) and Maneuverability (C_{10}).

The next stage involved the Fuzzy RIM approach, a novel methodology allowing us to compare a set of alternatives (four advanced trainer aircrafts) with respect to an ideal reference alternative combining quantitative and qualitative criteria. As a result of the process, the Alenia Aermacchi M-346 Master aircraft (A_2) of the Italian company Aermacchi is selected as the best option.

Moreover, a double sensitivity analysis is conducted to assess the robustness of the results. The aim of the first analysis is to verify the experts' judgments, whereas the goal in the second analysis is to compare the RIM approach with some MCDM methods. As such, the obtained results enhanced the consistency and robustness of the combination of the AHP and Fuzzy RIM techniques employed in this study.

To extend this work, a further study considering additional relevant alternatives, such as the Boeing-Saab T-X, which is currently under development, may be carried out. Moreover, future research could expand the scope of this paper by implementing the methodology through software which can solve real-world decision problems in any sphere of activity.

Declaration of competing interest

No author associated with this paper has disclosed any potential or pertinent conflicts which may be perceived to have impending conflict with this work. For full disclosure statements refer to https://doi.org/10.1016/j.asoc.2020.106061.

CRediT authorship contribution statement

J.M. Sánchez-Lozano: Funding acquisition, Investigation, Methodology, Supervision, Writing - review & editing. **O. Naranjo Rodríguez:** Formal analysis, Resources, Writing - original draft.

Table 14
Comparison of the alternatives based on FRIM/RIM approaches and the TOPSIS method.

Alternatives	FRIM/RIM Ranking	TOPSIS Ranking	WSM Ranking	Revised AHP Ranking
A_1.- KAI-T-50 Golden Eagle	2	1	2	2
A_2.- Alenia Aermacchi M-346 Master	1	2	1	1
A_3.- Yakovlev YAK-130	4	4	3	3
A_4.- Northrop F-5 Freedom Fighter	3	3	4	4

Acknowledgments

This work is partially supported by grants TIN2017-86647-P and PGC2018-097374-B-I00 from the Spanish Ministry of Economy and Competitiveness (MINECO) and the Spanish Ministry of Science, Innovation and Universities, respectively. J.M.S.L. also acknowledges the support of Grant 19882-GERM-15 from Fundación Séneca (Región de Murcia).

References

[1] J. Winchester, Modern Military Aircraft (The Aviation Factfile), Amber Books Ltd., London, 2004.
[2] Stockholm International Peace Research Institute (SIPRI), SIPRI yearbook 2019, in: Armaments, Disarmament and International Security, Oxford University Press, 2019.
[3] Ch. Ching-Hsue, Y. Kuo-Lung, H. Chia-Lung, Evaluating attack helicopters by AHP based on linguistic variable weight, European J. Oper. Res. 116 (1999) 423–435.
[4] I. De Leeneer, H. Pastijn, Selecting land mine detection strategies by means of outranking MCDM techniques, European J. Oper. Res. 139 (2002) 327–338.
[5] H. Pastijn, J. Leysen, Constructing an outranking relation with ORESTE, Math. Comput. Modell. 12 (10/11) (1989) 1255–1268.
[6] F.Y. Partovi, J.M. Epperly, A quality function deployment approach to task organization in peacekeeping force design, Socio-Econ. Plan. Sci. 33 (1999) 131–149.
[7] A.S. Wibowo, A. Erna-Permanasari, S. Fauziati, Combat aircraft effectiveness assessment using hybrid multi-criteria decision making methodology, in: 2nd International Conference on Science and Technology-Computer (ICST), IEEE, Yogyakarta, Indonesia, 2016.
[8] O. Adetunji, J. Bischoff, C.J. Willy, Managing system obsolescence via multicriteria decisión making, Syst. Eng. 21 (2018) 307–321.
[9] Y. Gazibey, O. Kantemir, A. Demirel, Interaction among the criteria affecting main battle tank selection: an analysis with DEMATEL method, Def. Sci. J. 65 (5) (2015) 345–355.
[10] T.L. Saaty, The Analytic Hierarchy Process, McGraw Hill International, 1980.
[11] P.J.M. Laarhoven, W. Pedrycz, A fuzzy extension of Saaty's priority theory, Fuzzy Sets and Systems 11 (1983) 229–241.
[12] C.G.E. Boender, J.G. de Graan, F.A. Lootsma, Multi-criteria decision analysis with fuzzy pairwise comparisons, Fuzzy Sets and Systems 29 (1989) 133–143.
[13] C.L. Hwang, K. Yoon, Multiple Attribute Decision Methods and Applications, Springer, Berlin Heidelberg, 1981.
[14] C.T. Chen, Extensions of the TOPSIS for group decision-making under fuzzy environment, Fuzzy Sets and Systems 114 (2000) 1–9.
[15] A. Mardani, A. Jusoh, K.M.D. Nor, Z. Khalifah, N. Zakwan, A valipour, multiple criteria decision-making techniques and their applications – a review of the literature from 2000 to 2014, Econ. Res.-Ekon. Istraž. 28 (1) (2015) 516–557.
[16] F.R.L. Junior, L. Osiro, L.C.R. Carpinetti, A comparison between fuzzy AHP and fuzzy TOPSIS methods to supplier selection, Appl. Soft Comput. 21 (2014) 194–209.
[17] M. Dagdeviren, S. Yavuz, N. Kılınç, Weapon selection using the AHP and TOPSIS methods under fuzzy environment, Expert Syst. Appl. 36 (2009) 8143–8151.
[18] T.C. Wang, T.H. Chang, Application of TOPSIS in evaluating initial training aircraft under a fuzzy environment, Expert Syst. Appl. 33 (2007) 870–880.
[19] J.M. Sánchez-Lozano, J. Serna, A. Dolón-Payán, Evaluating military training aircrafts through the combination of multi-criteria decision making processes with fuzzy logic. A case study in the Spanish Air Force Academy, Aerosp. Sci. Technol. 42 (2015) 58–65.
[20] E. Cables, M.T. Lamata, J.L. Verdegay, RIM-Reference Ideal Method in multicriteria decision making, Inform. Sci. 337–338 (2016) 1–10.
[21] S. Opricovic, G.H. Tzeng, The compromise solution by MCDM methods: A comparative analysis of VIKOR and TOPSIS, European J. Oper. Res. 156 (2004) 445–455.
[22] E. Cables, M.T. Lamata, J.L. Verdegay, in: Mikael Collan, Janusz Kacprzyk (Eds.), Soft Computing Applications for Group Decision-Making and Consensus Modeling: FRIM—Fuzzy Reference Ideal Method in Multicriteria Decision Making, Springer International Publishing AG, 2018.
[23] C.H. Cheng, D.L. Mon, Evaluating weapon system by analytical hierarchy process-based on fuzzy scales, Fuzzy Sets and Systems 63 (1994) 1–10.
[24] C.H. Cheng, D.L. Mon, Evaluating a weapon system using fuzzy analytical hierarchy process, Def. Sci. J. 44 (2) (1994) 165–172.
[25] D.L. Mon, C.H. Cheng, J.C. Lin, Evaluating weapon system using fuzzy analytic hierarchy process based on entropy weight, Fuzzy Sets and Systems 62 (2) (1994) 127–134.
[26] C.H. Cheng, Evaluating naval tactical missile systems by fuzzy AHP based on the grade value of membership function, European J. Oper. Res. 96 (1997) 343–350.
[27] C.H. Cheng, Y.-H. Liu, Y. Lin, Evaluating a weapon system using catastrophe series based on fuzzy scales, soft computing in intelligent systems and information processing, in: Proceedings of the 1996 Asian Fuzzy Systems Symposium, Kenting, Taiwan, 1996, pp. 212–217.
[28] C.H. Cheng, Evaluating weapon systems using ranking fuzzy numbers, Fuzzy Sets and Systems 107 (1) (1999) 25–35.
[29] C.H. Cheng, K.L. Yang, C.L. Hwang, Evaluating attack helicopters by AHP based on linguistic variable weights, European J. Oper. Res. 116 (1999) 423–435.
[30] J. Wang, K. Fan, Y. Su, Sh. Liang, W. Wang, Air combat effectiveness assessment of military aircraft using a fuzzy AHP and TOPSIS methodology, in: 2008 Asia Simulation Conference - 7th International Conference on System Simulation and Scientific Computing, Beijing, 2008, pp. 655–662.
[31] G. Arslan, Ö. Aydin, A new software development for fuzzy multicriteria decision-making, Technol. Econ. Dev. Econ. 15 (2) (2009) 197–212.
[32] K.P. Lin, K.C. Hung, An efficient fuzzy weighted average algorithm for the military UAV selecting under group decision-making, Knowl.-Based Syst. 24 (6) (2011) 877–889.
[33] W.M. Lu, T.C. Wang, A fuzzy multi-criteria model for the industrial cooperation program transaction strategies: A case in Taiwan, Expert Syst. Appl. 38 (3) (2011) 1490–1500.
[34] Y.J. Wang, T.C. Han, C.C. Chung, C.L. Fang, FAHP for selecting transport modes of Taiwan off-shore islands' military logistics, Int. J. Marit. Eng. 158 (4) (2016) 337–346.
[35] T.L. Saaty, Fundamentals of decision making and priority theory with the AHP RWS Publisher, Pittsburgh 1994.
[36] E.H. Forman, Random indices for incomplete pairwise comparison matrices, European J. Oper. Res. 48 (1990) 153–155.
[37] J.A. Alonso, M.T. Lamata, Consistency in the analytic hierarchy process: a new approach, Int. J. Uncertain. Fuzziness Knowl.-Based Syst. 14 (4) (2006) 445–459.
[38] L.A. Zadeh, Fuzzy sets, Inf. Control 8 (1965) 338–353.
[39] L.A. Zadeh, The concept of linguistic variable and its application to approximate reasoning, Inform. Sci. 8 (1975) 199–249.
[40] E. Triantaphyllou, Multi-Criteria Decision Making Methods: A Comparative Study, Kluwer Academic Publishers, Dordrecht, The Netherlands, 2000.
[41] S. Mahmoudi, A. Jalali, M. Ahmadi, P. Abasi, N. Salari, Identifying critical success factors in Heart Failure Self-Care using fuzzy DEMATEL method, Appl. Soft Comput. J. 84 (2019) 105729.
[42] G.J. Klir, B. Yuan, Fuzzy Sets and Fuzzy Logic: Theory and Applications, Prentice Hall, Upper Saddle River, NJ, 1995.
[43] Several Authors, Documento de Necesidad Operativa del Ejército del Aire, Sustitución del Sistema AE.09 (F-5), Technical Report, 2019, (only in Spanish).
[44] J.L. Joosung, Y. Hyungseok, A comparative study of technological learning and organizational capability development in complex products systems: Distinctive paths of three latecomers in military aircraft industry, Res. Policy 44 (2015) 1296–1313.
[45] I. Nicolin, B.A. Nicolin, Analysis of modern military jet trainer aircraft, Incas Bull. 10 (4) (2018) 193–202.
[46] KAI T-50 golden eagle, advanced trainer/ light attack, 2019, http://www.aeroflight.co.uk/aircraft/types/type-details/kai-t-50.htm [accessed 02.02.19].
[47] K.J. Hong, S.C.H. Hyeon, L. Ilwoo, Y.H. Don, $T-50/A-50$ FSD stores separation program, in: AIAA Atmospheric Flight Mechanics Conference and Exhibit, Keystone, Colorado, 2006.

[48] P. Collins, Flight test: KAI T-50 - Korea high, 2006, https://www.flightglobal.com/news/articles/flight-test-kai-t-50-korea-high-204780/ [accessed 02.02.19].
[49] ICD market research. The global military aircraft market 2011-2021, 2011, https://www.airforce-technology.com/projects/yak/ [accessed 03.02.19].
[50] S.p.A. Leonardo, M-346 master in training, air, trainers aircraft, 2019, https://www.leonardocompany.com/products/m-346?f=/air/aircraft/trainers [accessed 03.02.19].
[51] Flight International, Flight test: Alenia Aermacchi M-346 - one-stop warrior, 2006, https://www.flightglobal.com/news/articles/flight-test-alenia-aermacchi-m-346-one-stop-warrior-207881/ [accessed 03.02.19].
[52] V. Drushlyakov, The advanced aircraft of the Russian air force, Avion. World 5 (2003) http://www.yak.ru/ENG/FIRM/art_switch.php?art=1 [accessed: 04.02.19].
[53] K. Kubrynski, Conceptual and aerodynamic study of a highly manoeuvrable jet trainer, J. KONES Powertrain Transp. 23 (4) (2016).
[54] W. Gasich, Northrop F-5 Tactical Fighter in the Close Support Role, SAE Technical Paper 650236, 1965, pp. 1–13.
[55] M.S. Knaack, Encyclopedia of US air force aircraft and missile systems, in: Post-World War II Fighters, 1945-1973, Office of air force history, Washington, D.C., 1978.
[56] A.K. Noor, S.L. Venneri, Future Aeronautical and Space Systems, in: Prog. Astronaut. Aeronaut., vol. 72, American Institute of Aeronautics and Astronautics, Virginia, 1997.
[57] M.E. Eshelby, Aircraft Performance – Theory and Practice, Elsevier, London, 2000.
[58] Korea aerospace industries LTD, product T-50 family, 2019, http://www.koreaaero.com/english/product/fixedwing_t-50.asp [accessed: 03.03.19].
[59] J.V. Silió, Aviones Españoles del Siglo XX, Fundación Infante de Orleans Editor, Madrid, 2008.
[60] Defensa.com. La DGAM convocará un polémico concurso para comprar un avión que remplace a los C101 del Ejército del Aire, 2019, https://www.defensa.com/espana/dgam-convocara-inminentemente-polemico-concurso-para-comprar [accessed: 26.10.19].
[61] Los posibles sustitutos para el C-101 aviojet: aviones turbohélices y dos reactores italianos, 2019, http://www.outono.net/elentir/2019/08/19/ [accessed: 26.10.19].
[62] V. Belton, T.T. Gear, On a short-coming of Saaty's method of analytic hierarchies, Omega (1983) 228–230.
[63] T.L. Saaty, How to make a decision: The analytic hierarchy process, European J. Oper. Res. 48 (1) (1990) 9–26.
[64] J.M. Sanchez-Lozano, M.S. Garcia-Cascales, M.T. Lamata, C. Sierra, Decision Criteria for Optimal Location of Wind Farms. in Exploring Innovative and Successful Applications of Soft Computing, Information Science Publishing, Hershey, PA, 2013.
[65] J.M. Sánchez-Lozano, M.S. García-Cascales, M.T. Lamata, GIS-based onshore wind farm site selection using fuzzy multi-criteria decision making methods. Evaluating the case of southeastern Spain, Appl. Energy 171 (2016) 86–102.
[66] J.M. Sánchez-Lozano, M. Fernández-Martínez, M.T. Lamata, Near-earth asteroid impact dates: A reference ideal method (RIM) approach, Eng. Appl. Artif. Intell. 81 (2019) 157–168.